Mathematical Girls - Fermat's Last Theorem

C02

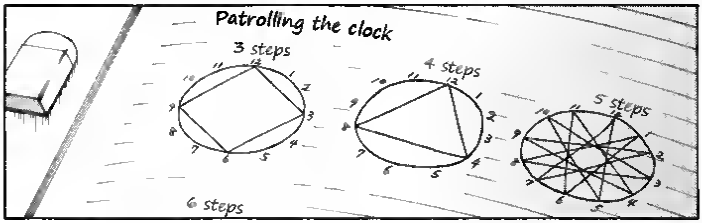
DKThias Scanlations
Eastern Atelier
Norway Scan
Forgotten Scans

Raw Provider : DKThias
Translator : Saiyaku
Proofreader : Volandum
Cleaner : DKThias
Redrawer : Flowersofmoe
Typesetter : Reconfigfin
Quality Checkers : Volandum
Saiyaku
DKThias
Flowersofmoe

#DKThias @irc.rizon.net
#Pocketloli @irc.irchighway.net
#Forgotten-Scans @irc.irchighway.net
Eastern Atelier

dkthias.com
fos-scans.com
easternatelierwordpress.com

HEY, IS THIS REALLY CONSIDERED MATHEMATICS?
IT'S AMUSING, JUST LIKE A GAME.


Patrolling the clock
3 steps
4 steps
5 steps
6 steps

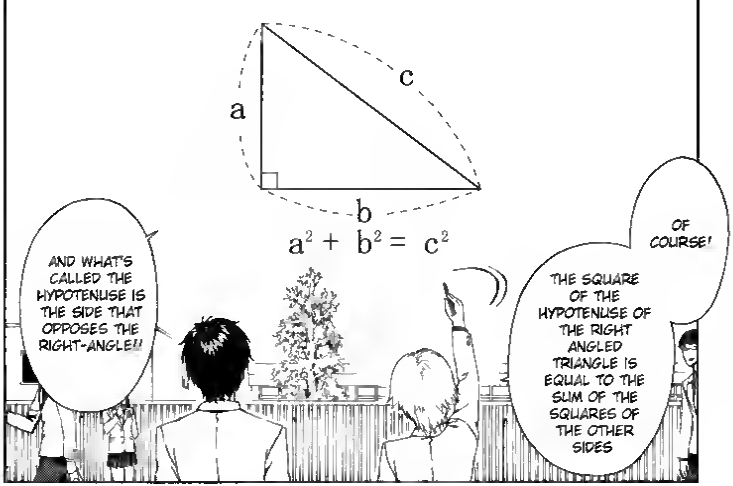
SENPAI?

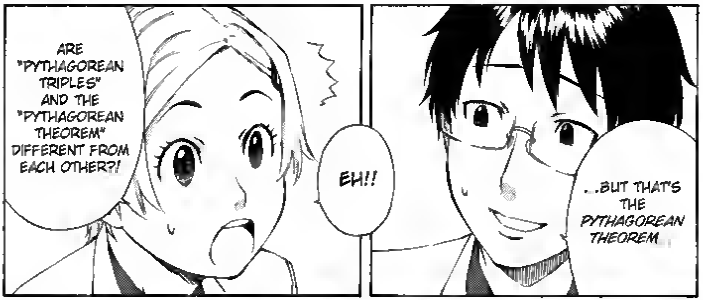
第 2 話
Chapter 2

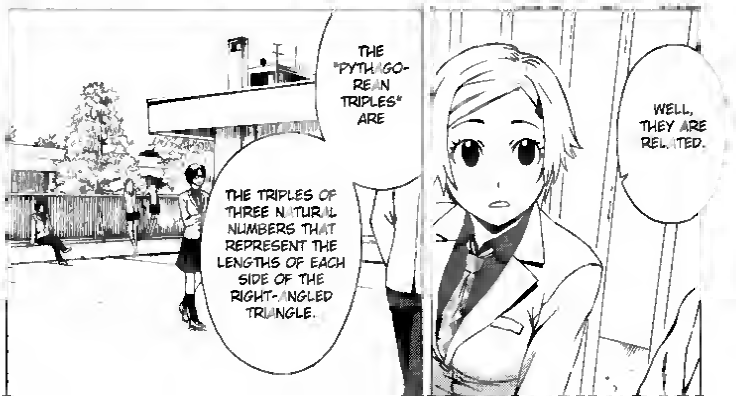
BA-DUMP
EH.
AH..
SORRY TO SURPRISE YOU, YOU WEREN'T RESPONDING AND
NO, IT'S FINE..
IS THAT SO?
I WAS THINKING ABOUT A RELATIVE OF MINE.

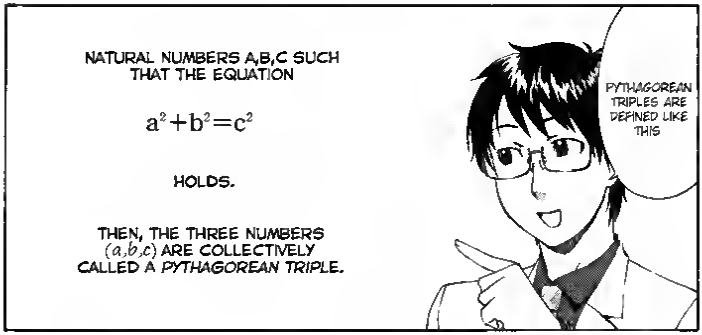
I usually teach her maths however
MUNCH
MUNCH
Tetra-chan is a first year high school student A Fouhai one year below me.
She is a girl I get along with and we study maths together.
sometimes she also surprises me with her bright ideas
OPENS WIDE
NOW THAT I REMEMBER, WHAT'S "MURAKI-SENSEI'S CARD"?
OH...
RUSTLE
RUSTLE
YEAH, THAT'S RIGHT

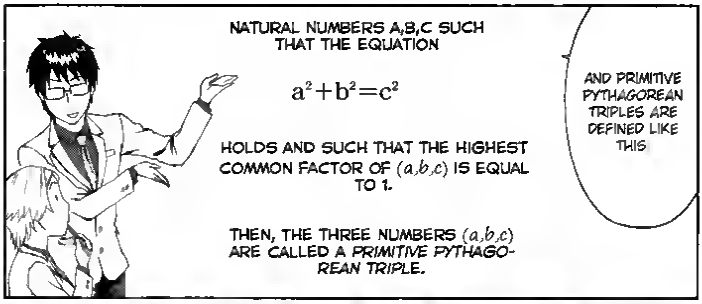
Are there infinitely many primitive Pythagorean triples?
SO, THAT'S WHAT WE GET FOR TODAY .
.....
BUT MURAKI-SENSEI IS SORT OF UNUSUAL.
HE MAKES QUESTIONS THAT HAVE NOTHING TO DO WITH CLASS EVERY TIME.
YEAH.
BY THE WAY TETRA-CHAN, DO YOU KNOW THOSE PRIMITIVE PYTHAGOREAN TRIPLES?


a
c
b
$a^2 + b^2 = c^2$
AND WHAT'S CALLED THE HYPOTENUSE IS THE SIDE THAT OPPOSES THE RIGHT-ANGLE!!
OF COURSE!
THE SQUARE OF THE HYPOTENUSE OF THE RIGHT ANGLED TRIANGLE IS EQUAL TO THE SUM OF THE SQUARES OF THE OTHER SIDES


ARE "PYTHAGOREAN TRIPLES" AND THE "PYTHAGOREAN THEOREM" DIFFERENT FROM EACH OTHER?!
EH!!
..BUT THAT'S THE *PYTHAGOREAN THEOREM*


THE "PYTHAGOREAN TRIPLES" ARE
THE TRIPLES OF THREE NATURAL NUMBERS THAT REPRESENT THE LENGTHS OF EACH SIDE OF THE RIGHT-ANGLED TRIANGLE.
WELL, THEY ARE RELATED.

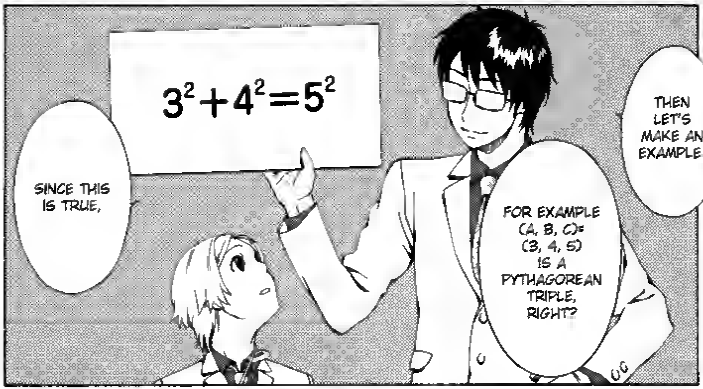

NATURAL NUMBERS A,B,C SUCH THAT THE EQUATION
$a^2+b^2=c^2$
HOLDS.
THEN, THE THREE NUMBERS (a,b,c) ARE COLLECTIVELY CALLED A PYTHAGOREAN TRIPLE.
PYTHAGOREAN TRIPLES ARE DEFINED LIKE THIS

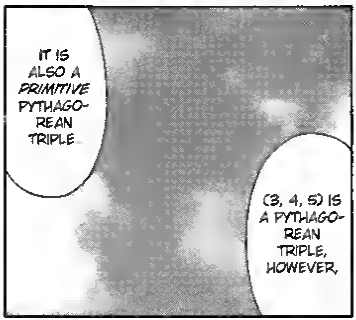

NATURAL NUMBERS A,B,C SUCH THAT THE EQUATION
$a^2+b^2=c^2$
HOLDS AND SUCH THAT THE HIGHEST COMMON FACTOR OF (a,b,c) IS EQUAL TO 1.
THEN, THE THREE NUMBERS (a,b,c) ARE CALLED A PRIMITIVE PYTHAGOREAN TRIPLE.
AND PRIMITIVE PYTHAGOREAN TRIPLES ARE DEFINED LIKE THIS

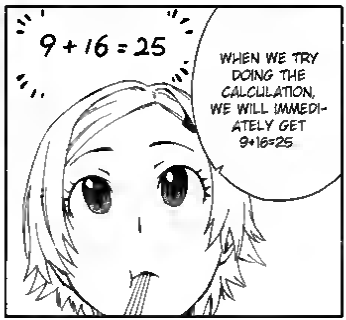
THAT SET OF THREE NUMBERS IS ALSO A PRIMITIVE PYTHAGOREAN TRIPLE.
MUNCH
MUNCH
AND WHEN THE HIGHEST COMMON FACTOR IS 1,
THAT'S TO SAY, WHEN THE THREE SIDES OF THE RIGHT ANGLED TRIANGLE ARE NATURAL NUMBERS,
THAT TRIPLE IS A SET OF PYTHAGOREAN NUMBERS

...SENPAI, I
DON'T UNDERSTAND THE MEANING OF THE HIGHEST COMMON FACTOR IS I VERY WELL YET...

IS FOR MURAKI-SENSEI'S PROBLEM, WHETHER THE PRIMITIVE PYTHAGOREAN TRIPLES LIKE THAT ARE INFINITE OR NOT,
IS WHAT THIS QUESTION IS ABOUT.


$3^2+4^2=5^2$
SINCE THIS IS TRUE,
FOR EXAMPLE (A, B, C)= (3, 4, 5) IS A PYTHAGOREAN TRIPLE, RIGHT?
THEN LET'S MAKE AN EXAMPLE.


IT IS ALSO A PRIMITIVE PYTHAGOREAN TRIPLE.
(3, 4, 5) IS A PYTHAGOREAN TRIPLE, HOWEVER,


9+16=25
WHEN WE TRY DOING THE CALCULATION, WE WILL IMMEDIATELY GET 9+16=25

THE HIGHEST COMMON FACTOR OF 3, 4, 5——
IN OTHER WORDS, THE LARGEST NUMBER
THOSE THREE NUMBERS CAN BE DIVIDED BY WITHOUT A REMAINDER, IS 1.
...I'M SORRY.
I STILL DON'T GET THE DIFFERENCE BETWEEN THE PYTHAGOREAN TRIPLES AND THE PRIMITIVE PYTHAGOREAN TRIPLES...
IT'S OKAY.
LET'S TALK A LITTLE MORE ABOUT THE EXAMPLE.
(3, 4, 5) IS A PYTHAGOREAN TRIPLE,
AS WELL AS A PRIMITIVE PYTHAGOREAN TRIPLE.
BREAD ROLL
HOWEVER, HOW ABOUT (6, 8, 10) WHICH WE GET BY MULTIPLYING EACH OF THOSE THREE NUMBERS BY TWO?
THAT IS ANOTHER PYTHAGOREAN TRIPLE, BUT IT'S NOT A PRIMITIVE PYTHAGOREAN TRIPLE.

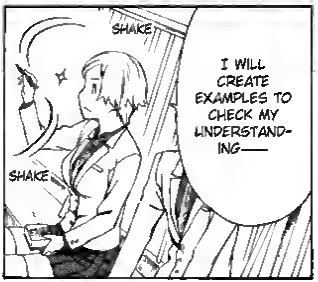
$6^2+8^2=10^2$
CERTAINLY TRUE
$6^2=36$
$8^2=64$
$10^2=100$
RIGHT?
SO, 36+64=100 IS...
MUNCH
(6, 8, 10) IS NOT A PRIMITIVE PYTHAGOREAN TRIPLE..
SINCE THE HIGHEST COMMON FACTOR OF (6, 8, 10) IS 2,
UMM... HOWEVER,
SO WE CAN SAY THAT (6, 8, 10) IS A PYTHAGOREAN TRIPLE
IS WHAT'S BEING QUESTIONED.
YEAH.
FOR MURAKI-SENSEI'S PROBLEM, THE EXISTENCE OF INFINITELY MANY OF THOSE PRIMITIVE PYTHAGOREAN TRIPLES
THE PRIMITIVE PYTHAGOREAN TRIPLES ARE THOSE WHERE THE THREE NUMBERS CAN ONLY BE DIVIDED BY 1 WITHOUT A REMAINDER...
THAT'S WHAT IT MEANS, RIGHT?

SENPAI, THAT'S WEIRD!
BETWEEN THE THREE SIDES A, B, C OF A RIGHT-ANGLED TRIANGLE, THERE IS ALWAYS THE RELATIONSHIP $A^2+B^2=C^2$, RIGHT?
SULK
AND SINCE, IF WE CHANGE THE LENGTHS OF THE SIDES TO VARIOUS LENGTHS WE CAN CREATE AN INFINITE NUMBER OF RIGHT ANGLED TRIANGLES,
ISN'T IT ONLY NATURAL THAT THERE ARE INFINITELY MANY PRIMITIVE PYTHAGOREAN TRIPLES ?
CALM DOWN AND TRY TO THINK ABOUT THE CONDITIONS ON PRIMITIVE PYTHAGOREAN TRIPLES.
E...
AH!!
SHAKE
SHAKE
WRONG WRONG WRONG WRONG WRONG WRONG WRONG!!

I FORGOT IT!!
THE REQUIRE-MENT FOR "NATURAL NUMBERS"!
IF WE ARE GOING TO TACKLE THIS PROBLEM, THEN HOW ABOUT WE START
BY FINDING MORE EXAMPLES OF PRIMITIVE PYTHAGOREAN TRIPLES LIKE (3, 4, 5)?
THAT'S RIGHT.
WE CAN FREELY CHOOSE TWO OUT OF THE THREE SIDES, SO WE CAN MAKE THEM NATURAL NUMBERS
BUT THEN, IT'S NOT A GIVEN THAT THE REMAINING SIDE, TOO, WILL BE A NATURAL NUMBER

EXAMPLES ARE THE TOUCH-STONE FOR UNDER-STANDING,
RIGHT?
CLICK
UNDERSTOOD IT'S LIKE WHAT SENPAI ALWAYS SAYS

SHE'S REALLY A SINCERE AND LIVELY GIRL.
.BUT
stick
stick
stick
THE CONDITION-FORGETTING TETRA-CHAN IS DOING WELL, I GUESS


SHAKE
SHAKE
I WILL CREATE EXAMPLES TO CHECK MY UNDERSTAND-ING——

2—A

Oh!
rice
...HEY, PLEASE REFRAIN FROM SWINGING THE CHOPSTICKS, IT'S DANGEROUS.
ADJUSTS GLASSES

WHERE HAVE YOU BEEN?
I WAS... ON THE ROOFTOP...
The surrounding atmosphere tenses up suddenly with her very approach.
Miruka-san is a talented girl in her second year of high school in the same class as me
She is outstandingly good at mathematics.

HAVING LUNCH?
ON THE ROOFTOP.
STARE
YE...AH
LI... UMM.
KEEPING IT FROM ME?
HUMPH..

C-COME ON, MIRUKA-SAN, YOU WEREN'T IN THE CLASSROOM DURING THE LUNCH BREAK, WERE YOU?
SO, I THOUGHT YOU WERE WITH AY-AY...
RIGHT?
WHAT EXACTLY AM I MAKING EXCUSES FOR.
But ..
I can't hold my head up before Miruka-san
How should I put it...
THEN
I WAS IN THE STAFF ROOM

I WENT TO SHOW THAT REPORT FROM EARLIER TO MURAKI-SENSEI.
GRAB
JUST LIKE ALWAYS, THERE WAS A NEW CARD.
IT'S A STRANGE QUESTION.
"Muraki-sensei" is our maths teacher
He is quite eccentric, but he likes us,
and gives us mathematical problems of great interest
It's really refreshing when he comes up with problems that have nothing to do with our lessons or exams.

ON A UNIT CIRCLE WHOSE CENTER IS THE ORIGIN, DO INFINITELY MANY RATIONAL POINTS EXIST?

NOD

THAT'S RIGHT.

A RATIONAL NUMBER, LIKE 1/2 OR -2/5, IS A NUMBER THAT EXPRESSES A RATIO BETWEEN INTEGERS.

AND POINTS WITH RATIONAL NUMBERS AS COORDINATES ARE CALLED RATIONAL POINTS.

A RATIONAL POINT IS ...

A POINT SUCH THAT THE X COORDINATE AND Y COORDINATE ARE BOTH RATIONAL NUMBERS.

SPIN

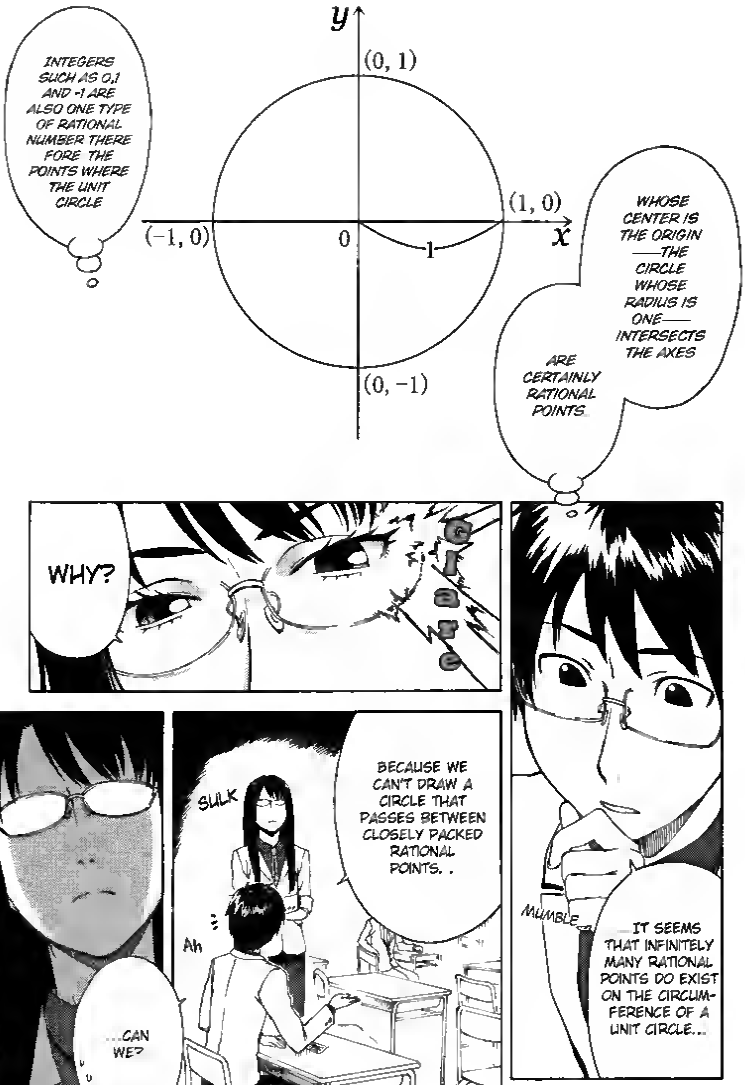
ON THE CIRCUMFERENCE OF A UNIT CIRCLE WHOSE CENTER IS THE ORIGIN,

$(1, 0), (0, 1)$

$(-1, 0), (0, -1)$

THERE ARE THE SELF-EVIDENT RATIONAL POINTS, NAMELY (1, 0), (0, 1), (-1, 0) AND (0, -1)

AND THEN, BESIDES THESE, DO *INFINITELY MANY* RATIONAL POINTS EXIST? THAT'S WHAT THIS PROBLEM IS ASKING.


INTEGERS SUCH AS 0,1 AND -1 ARE ALSO ONE TYPE OF RATIONAL NUMBER THERE FORE THE POINTS WHERE THE UNIT CIRCLE
y
(0, 1)
(1, 0)
(−1, 0)
0
1
x
(0, −1)
WHOSE CENTER IS THE ORIGIN ——THE CIRCLE WHOSE RADIUS IS ONE—— INTERSECTS THE AXES
ARE CERTAINLY RATIONAL POINTS
WHY?
Gasp
MUMBLE
...IT SEEMS THAT INFINITELY MANY RATIONAL POINTS DO EXIST ON THE CIRCUMFERENCE OF A UNIT CIRCLE...
SULK
Ah
BECAUSE WE CAN'T DRAW A CIRCLE THAT PASSES BETWEEN CLOSELY PACKED RATIONAL POINTS. .
...CAN WE?

SURPRISE
THAT'S NOT MATHS.
IN THE REAL WORLD, NO MATTER HOW ACCURATELY WE DRAW A CIRCLE, WE CAN'T TELL SOMETHING LIKE WHETHER WE PASS RATIONAL POINTS
NEITHER OUR HANDS, NOR A COMPASS,
HOWEVER, WE
POSSESS A TOOL FAR SUPERIOR TO ANYTHING THAT EXISTS IN THE REAL WORLD
CAN DRAW A TRUE CIRCLE.

WE POSSESS MATHEMATICS.
DON'T YOU THINK SO?
...I GET IT, MIRUKA-SAN.
I ENDED UP SAYING SOMETHING IRRESPONSIBLE
ANYWAY, BECAUSE OF THE CONDITION THAT COORDINATES OF POINTS
CAN BE EXPRESSED AS $\left(\frac{a}{b}, \frac{c}{d}\right)$ SUCH THAT a, b, c, AND d ARE INTEGERS, AND THEY ARE ON THE CIRCUMFERENCE OF THE UNIT CIRCLE,
IF WE DO SOME CALCULATIONS ON PAPER, WE WILL KNOW RIGHT?

From its prime factors, the structure of an integer is known.
HMM.
CERTAINLY. THAT WON'T BE BAD EITHER
By its integer ratio, a rational number's is shown.

WONDERING IF YOU HAVE BEEN HAVING LUNCH ALONE,
FOR EXAM-PLE.
EH?!
Surprised
OTHER STUFF?
HOWEVER, I HAVE BEEN THINKING ABOUT OTHER STUFF...
OR POSSIBLY, WHETHER THE RATIONAL NUMBERS ON THE CIRCLE'S CIRCUMFERENCE COULD CORRESPOND TO SOMETHING INFINITE,
RIP
OR SOME-THING.
...I WAS HAVING LUNCH ON THE ROOFTOP TOGETHER WITH TETRA-CHAN.

I don't know what's going on any more
YOU ARE AN HONEST PERSON
I GRANT THEE A KNIGHT'S TITLE AND SWORD

Yuuki Hiroshi's extra lectures 2

Integers and rational numbers.

Numbers have names. We call the numbers 1,2,3... "natural numbers". 0 and -1 are not natural numbers. And we call the numbers ...-3,-2,-1,0,1,2,3... "integers". Natural numbers are all integers. 0.5 and $\frac{1}{3}$ are not integers.

We call a number formed by dividing an integer by a natural number a "rational number". Since 0.5 can be represented as $\frac{1}{2}$ it's a rational number. $\frac{1}{3}$ and $-\frac{2}{5}$ are also rational numbers. 5 can be represented as $\frac{5}{1}$, so it's also a rational number. 0 too, can be represented as $\frac{0}{1}$, so it's also a rational number. We can say that both natural numbers and integers are entirely rational numbers.

The circular constant pi, 3.14159... is not a rational number, since it can't be represented as "a number formed by the division of an integer by a natural number".

The problem about primitive Pythagorean triples that was written on Tetra-chan's card is a problem about natural numbers, and that one written in Miruka-san's card is a problem about rational numbers.

When thinking about mathematics, it's important to be aware what sorts of numbers the problem is dealing with.